Function of a Function

If we have a function $f(x)$ and a function $g(x)$ then $fg(x)$ is a function of a function. It's best explained with an example.

Example

$f(x) = 4x - 1$, $g(x) = x+1$ what is $fg(x)$?

What we have to be careful of is the order, we work from the left, adding brackets might help $fg(x) = f(g(x))$

We know $f(x) = 4x - 1$, therefore,

$f(g(x)) = 4(g(x)) - 1$ we just write $g(x)$ instead of x.

Now we know what $g(x)$ is, $g(x) = x + 1$ so fill that in:

$$f(g(x)) = 4(x+1) - 1$$

Now just expand and simplify:

$$f(g(x)) = 4(x+1) - 1 = 4x + 4 - 1 = 4x + 3$$

$$f(g(x)) = fg(x) = 4x+3$$

Now a harder example,

$\boxed{f(x) = 5(3x-1)}$, $\boxed{g(x) = 3(2x+5)}$ what is $gf(x)$?

so f and g are the opposite way round now so we have $g(f(x))$

$$g(x) = 3(2x+5) \Rightarrow g(f(x)) = 3(2(f(x))+5)$$

$$\Rightarrow g(f(x)) = 3(2(5(3x-1))+5) = 3(2(15x-5)+5) = 3(30x-10+5)$$

$$= 3(30x-5) = \underline{90x-15} \Rightarrow \boxed{gf(x) = 90x-15}$$

Function of itself

This is the same as function of a function but we use the same equation twice:

Example

$f(x) = 4(3x-1)$, what is $ff(x)$?

As before $ff(x) = f(f(x))$

$f(x) = 4(3x-1)$ so $f(f(x)) = 4(3(f(x))-1)$

$f(f(x)) = 4(3(4(3x-1))-1) = 4(3(12x-4)-1)$

$= 4(36x - 12 - 1) = 4(36x-13)$

$= 144x - 52$

$$\boxed{ff(x) = 144x - 52}$$

Note: When you have lots of brackets like above remember:

1) You must have as many)'s as you have ('s so if you don't you've lost or gained one.

2) Work outwards from the inner most brackets.