Sequences

There are 3 types of sequences; arithmetic, quadratic and geometric. You need to be able to identify which type a sequence is:

Arithmetic sequences increase by a constant amount from term to term.

e.g. $3, \ 5, \ 7, \ 9, \ 11$

$+2 \quad +2 \quad +2 \quad +2$

To solve for the n^{th} term we know in this example it will be of the form $2n + c$.

We get the 2 from the difference between each term (above). To find c, we know the first term $(n = 1)$ is 3. Therefore,

$$2 \times 1 + c = 3 \Rightarrow c = 3 - 2 = +1$$

So the n^{th} term is: $\boxed{a_n = 2n + 1}$ $\Rightarrow$ 1st term: $(2 \times 1) + 1 = 3$

2nd term: $(2 \times 2) + 1 = 5$

3rd term: $(2 \times 3) + 1 = 7$

etc.

Quadratic Sequence the difference between each term increases by a constant amount each time.

e.g. $5, \ 14, \ 27, \ 44, \ 65, \ 90$

$+9 \quad +13 \quad +17 \quad +21 \quad +25$

$+4 \quad +4 \quad +4 \quad +4$

The value of the second difference (+4 above) tells us the value of the 'n^2' coefficent:

Second difference :	2	4	6	8	a
Term :	n^2	$2n^2$	$3n^2$	$4n^2$	$\frac{an^2}{2}$

So in our example the second difference was $+4$ so we have $2n^2$ term. Then we work out what is left:

n	1	2	3	4	5	6	
Sequence	5	14	27	44	65	90	① Sequence given to us
$2n^2$	2	8	18	32	50	72	② Sequence of $2n^2$
Difference	3	6	9	12	15	18	① - ②

$+3 \quad +3 \quad +3 \quad +3 \quad +3$

Now find the n^{th} term of the difference

↳ so $3n + c$

$3 \times 1 + c = 3 \Rightarrow c = 0$ } so n^{th} term of difference is $3n$.

So the n^{th} term of our sequence is the two added together:

$$\boxed{a_n = 2n^2 + 3n \ (+0)}$$

Geometric sequences are multiplied by a constant factor to get to the next term.

e.g. $\boxed{24}$, 48, 96, 192, 384

$\times 2 \quad \times 2 \quad \times 2 \quad \times 2$

The n^{th} term of a geometric sequence is:

$$u_n = a_1 r^{n-1}$$

first sequence term. common factor (2 above)

So the n^{th} term of the above is:

$$u_n = 24 \times 2^{n-1}$$

But this can be simplified as 24 has factors of 2:

$$24 = 12 \times 2 = 6 \times 2 \times 2 = 3 \times 2 \times 2 \times 2 = 3 \times 2^3$$

So, $u_n = 3 \times 2^3 \times 2^{n-1} = 3 \times 2^{n-1+3} = \boxed{3 \times 2^{n+2}}$

by the laws of indices