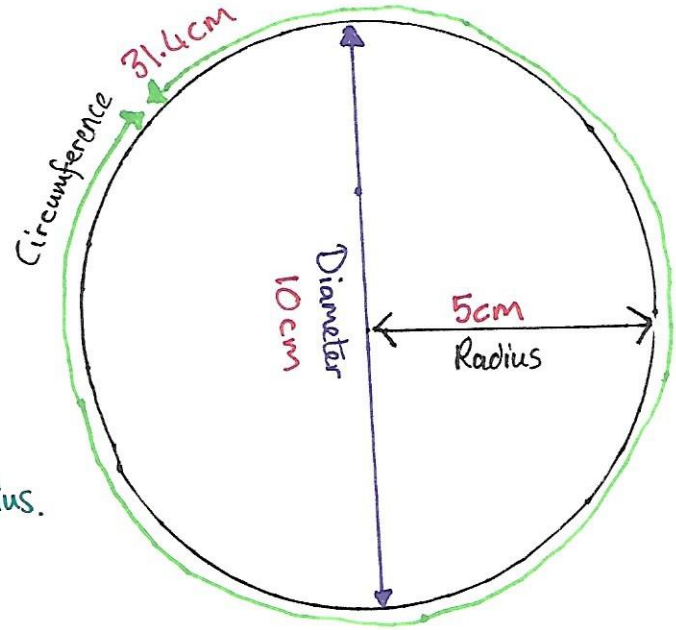
Circles



Radius: The distance from the centre to the edge of a circle.

5cm in our circle.

Diameter: The distance from one edge to the opposite edge (passing through the centre). This is twice the radius.

$D = 2r$

$2 \times 5cm = 10cm$ in our circle.

Circumference: The distance around the circle, or the perimeter. This is the radius multiplied by 2π. $\pi = 3.14159265...$

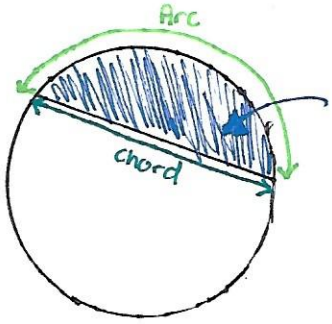
$C = 2\pi r$

$2\pi \times 5cm = 31.4cm$ in our circle.

Area: The amount of surface inside the circle. This is the radius squared multiplied by pi.

$A = \pi r^2$

$\pi \times (5cm)^2 = \pi \times 25\ cm^2 = 78.5\ cm^2$ in our circle.

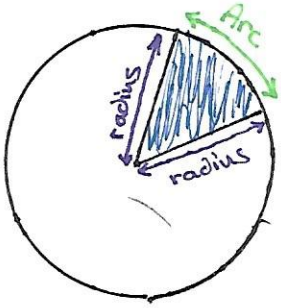


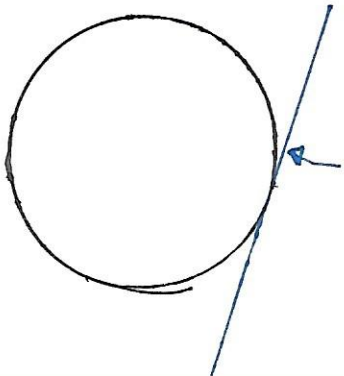
Segment: Is an area enclosed by an arc and a chord.

Arc: Is part of the circumference.

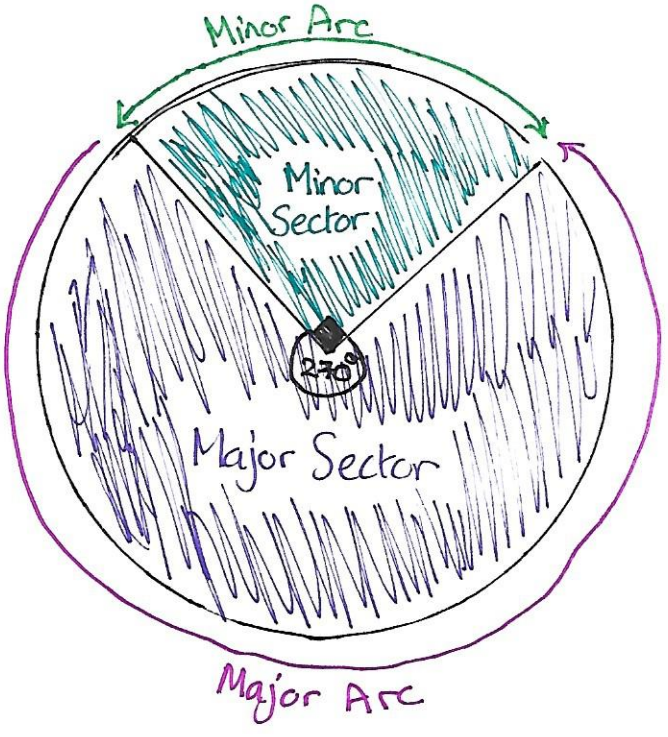
Chord: is a straight line joining two points on the circumference of a circle.

Sector: An area enclosed by two radii and an arc. It's a fraction of the area and looks like a pizza slice.





Tangent: A straight line which touches the edge of the circle. It has the same gradient as the point it touches.



The smaller sector is called the minor sector, the larger sector is called the major sector.

The smaller arc is called the **minor arc**, the larger arc is called the major arc.

There are 360 degrees in a circle so the angle in the minor sector added to the angle in the major sector must equal 360°.

Minor sector angle + Major sector angle = 360°.

$90° + 270° = 360°$ in our circle.

Area of a sector is a fraction of the area of the whole circle found using the angle of the sector.

$$\text{Area of sector} = \frac{\text{Sector angle}}{360°} \times \underbrace{\pi r^2}_{\text{Area of circle.}}$$

$$\text{Area of our minor sector} = \frac{90°}{360°} \times \pi \times (5\text{cm})^2 = 19.6\text{cm}^2$$

$$\text{Area of our major sector} = \frac{270°}{360°} \times \pi \times (5\text{cm})^2 = 50.9\,\text{cm}^2$$

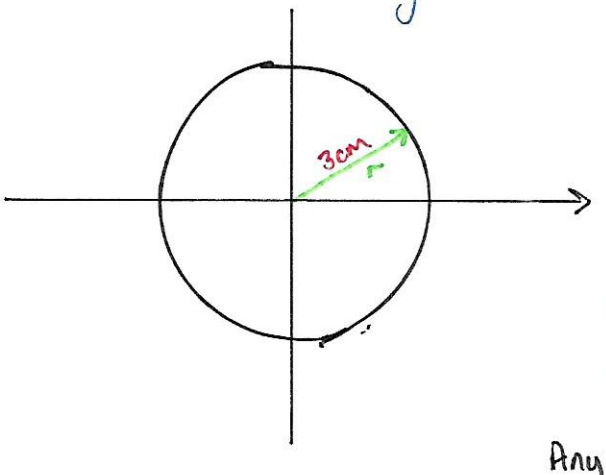
Length of an arc: is a fraction of the circumference of the circle found by using the angle of the sector.

$$\text{Length of arc} = \frac{\text{Angle of sector}}{360^\circ} \times \underbrace{2\pi r}_{\text{circumference}}$$

Length of our minor arc $= \frac{90^\circ}{360^\circ} \times 2\pi \times 5 = 7.85$ cm

Length of our major arc $= \frac{270^\circ}{360^\circ} \times 2\pi \times 5 = 23.56$ cm

Equation of a circle



The equation of a circle centered at (0,0) is the sum of x^2 and y^2 is equal to the radius of the circle squared.

Equation of a circle: $x^2 + y^2 = r^2$

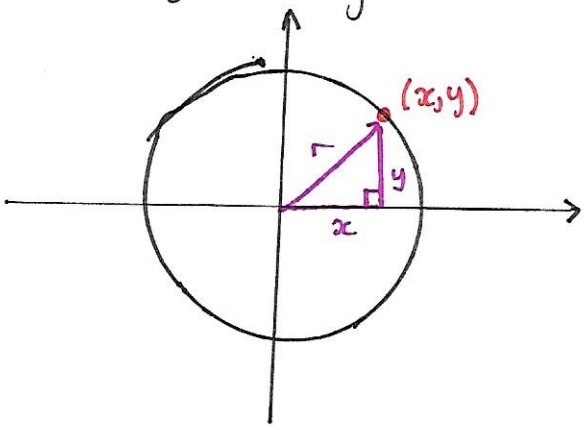
for our circle $x^2 + y^2 = 9$

Any co-ordinate (x,y) that satisfied this equation is on the circumference of the circle.

This is just an example of pythagorus theorem:

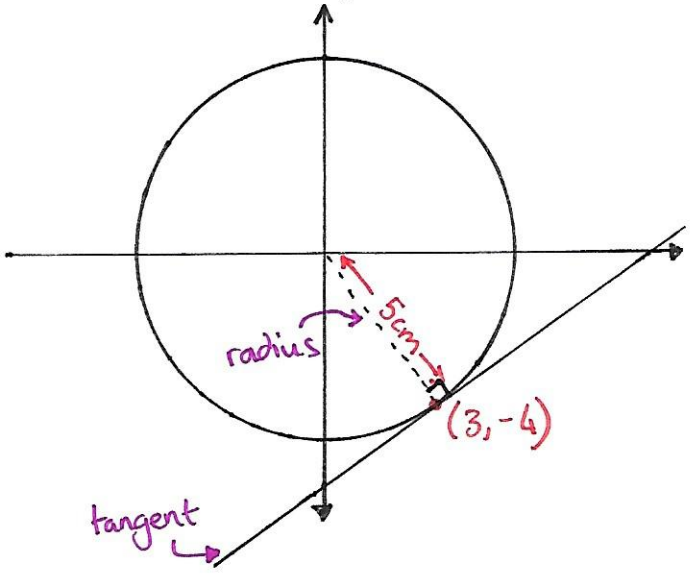
$a^2 + b^2 = c^2$

for a random point of the circumference (x,y) we can make a right angle triangle and so apply Pythagorus theorem.



Equation of a tangent to a circle

Any straight line has the form $y = mx + c$ so first we need to find the gradient 'm'.



If we consider our example (left), the radius is 5cm so the equation of the circle is $x^2 + y^2 = 25$.

We know the gradient of the radius touching the tangent:

$$\text{gradient of radius} = \frac{\text{difference in } y}{\text{difference in } x} = \frac{-4}{3}$$

The gradient of this radius is $-\frac{4}{3}$. The tangent is perpendicular to the radius and so the gradient of the tangent is the negative reciprocal of the radius gradient.

Negative reciprocal is 1 over and then swap the sign..

e.g. $x \xrightarrow{\text{negative reciprocal}} -\frac{1}{x}$

Returning to our example:

Negative reciprocal of $-\frac{4}{3}$ is $-\left(\frac{1}{-4/3}\right) = \frac{3}{4}$.

So the gradient of the tangent is $3/4$.

So our equation of the tangent is $y = \frac{3}{4}x + c$.

To find c, we know the point $(3,-4)$ is on the tangent so sub that in:

$$-4 = \frac{3}{4} \times 3 + c \quad \Rightarrow \quad -4 = \frac{9}{4} + c$$

$$\hookrightarrow \quad c = -4 - \frac{9}{4} = -\frac{25}{4}$$

Just put this in the calculator if you're unsure.

So the equation of the tangent is:

$$y = \frac{3}{4}x - \frac{25}{4}$$