

Logarithms

A **logarithm** tells us how many times we must multiply a number to get another. For example, $2 \times 2 \times 2 = 8$ or to write this more compactly $2^3 = 8$.

So we consider our question, how many times must we multiply 2 by itself to get 8. We can see clearly above that the answer is 3. We write this in log form as

$$\log_2(8) = 3$$

↑ ↑
base exponent

In general, if: $a^b = c$, then $\log_a(c) = b$.

There are two common types of logarithm:

1) **Base 10**, as this is very common we write just $\log()$ instead of $\log_{10}()$, they are equivalent.

2) **Base e** , this is used a lot in science and maths and it is called the **natural log** written as $\ln()$.
Euler's number
= 2.71828...

Rules of Logarithms

First Law: $\log A + \log B = \log AB$

For example $\log_{10} 5 + \log_{10} 4 = \log_{10} (5 \times 4) = \log_{10} 20$
But be careful. This law is only true if the base

is the same on both log.

$$\log_5 5 + \log_{10} 4 \neq \log 20$$

Second Law: $\log A - \log B = \log \frac{A}{B}$

For example $\log_{10} 12 - \log_{10} 4 = \log_{10} \frac{12}{4} = \log_{10} 3$

As above this only applies to logs with the same base.

Third Law: $\log A^n = n \log A$

For example $\log 8 = \log 2^3 = 3 \log 2$

Additional truths:

$$\log 1 = 0$$

This is true regardless of the base because any number to the power of 0 is 1.

$$x^0 = 1$$

$$\log_m m = 1$$

Clearly any number to the power of one is that number.

$$1^1 = 1$$

$$2^1 = 2$$

$$3^1 = 3$$

etc.