Matrices

A matrix is an array of numbers in rows and colums that is treated as a single entity.

Adding matrices

To add twomatrices is very simple, simply add the same point in each matrix;

e.g. $\begin{pmatrix} 3 & -2 \\ 5 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 7 \\ 8 & 0 \end{pmatrix} = \begin{pmatrix} 3+1 & -2+7 \\ 5+8 & 4+0 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 13 & 4 \end{pmatrix}$

Subtracting matrices

Similar to above we just subtract the same point in the matrix.

e.g. $\begin{pmatrix} 3 & -2 \\ 5 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 7 \\ 8 & 0 \end{pmatrix} = \begin{pmatrix} 3-1 & -2-7 \\ 5-8 & 4-0 \end{pmatrix} = \begin{pmatrix} 2 & -9 \\ -3 & 4 \end{pmatrix}$

Multiplying matrices

This is slightly more complicated. Here, ~~we~~ multiply and sum a whole row by a whole column. For the top left number, it'sinthe 1^{st} row and the 1^{st} column so we'd take the 1^{st} row of the left matrix and the 1^{st} column of the right matrix. The fully multiplied matrix has the form:

2x2 matrix: $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$

3×3 matrix: $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \times \begin{pmatrix} j & k & l \\ m & n & o \\ p & q & r \end{pmatrix} = \begin{pmatrix} aj+bm+cp & ak+bn+cq & al+bo+cr \\ dj+em+fp & dk+en+fq & dl+eo+fr \\ gj+hm+ip & gk+hn+iq & gl+ho+ir \end{pmatrix}$

This first matrix contributes rows. Which row we use depends the row of the number we are working out in the answer matrix.

The second matrix contributes columns. Which column we use depends on the column of the number we are working out in the answer matrix.

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \times \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

This is in the 2nd row and the third column so we take the 2nd row from the left matrix and the 3rd column from the right matrix.

Then we time the first numbers in each, and the second etc.

$$\begin{pmatrix} d & e & f \end{pmatrix} \quad \begin{pmatrix} l \\ o \\ r \end{pmatrix} \Rightarrow \underbrace{dl + eo + fr}$$

This is the value of the number in the 2nd row, 3rd column of the answer matrix.

Examples

1) $\begin{pmatrix} 3 & -2 \\ 5 & 4 \end{pmatrix} \times \begin{pmatrix} 1 & 7 \\ 8 & 0 \end{pmatrix} = \begin{pmatrix} (3\times1)+(-2\times8) & (3\times7)+(-2\times0) \\ (5\times1)+(4\times8) & (5\times7)+(4\times0) \end{pmatrix}$

$= \begin{pmatrix} 3-16 & 21+0 \\ 5+32 & 35+0 \end{pmatrix} = \begin{pmatrix} -13 & 21 \\ 37 & 35 \end{pmatrix}$

2) $\begin{pmatrix} 3 & 1 & 0 \\ -2 & 2 & 1 \\ 0 & -3 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 2 \\ 0 & -3 & 1 \\ 3 & -1 & 1 \end{pmatrix} = \begin{pmatrix} (3\times1)+(1\times0)+(0\times3) & (3\times2)+(1\times-3)+(0\times-1) & (3\times2)+(1\times1)+(0\times1) \\ (-2\times1)+(2\times0)+(1\times3) & (-2\times2)+(2\times-3)+(1\times-1) & (-2\times2)+(2\times1)+(1\times1) \\ (0\times1)+(-3\times0)+(1\times3) & (0\times2)+(-3\times-3)+(1\times-1) & (0\times2)+(-3\times1)+(1\times1) \end{pmatrix}$

$= \begin{pmatrix} 3+0+0 & 6+(-3)+0 & 6+1+0 \\ -2+0+3 & -4-6-1 & -4+2+1 \\ 0+0+3 & 0+9-1 & 0-3+1 \end{pmatrix} = \begin{pmatrix} 3 & 3 & 7 \\ 1 & -11 & -1 \\ 3 & 8 & -2 \end{pmatrix}$