(19) All cubic graphs look like this:

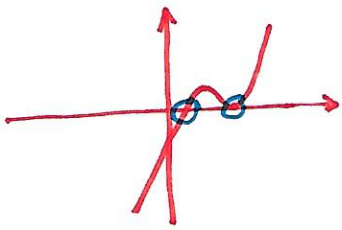
A graph crosses the x-axis the same number of times as how many roots it has:

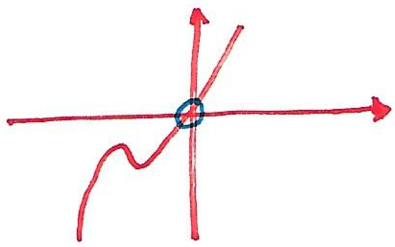
(i) Needs to cross x-axis twice

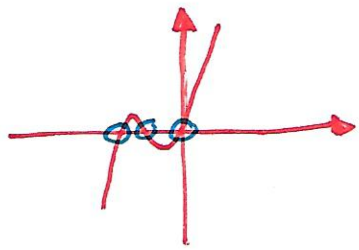
(ii) Needs to cross x-axis once

(iii) Needs to cross x-axis 3 times

e.g.







(20) The equation doesn't factorise easily so we just use the quadratic formula:

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \text{for equation} \quad ax^2+bx+c=0}$$

So for $2x^2+2x-3=0$,

$a=2$, $b=2$ and $c=-3$

$$x = \frac{-2 \pm \sqrt{2^2-4\times 2\times -3}}{2\times 2}$$

just put it straight in your calculator.

$= 0.823$ or -1.82

(21) Sorry this is really an A-level question but it's not too difficult if you understand the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

This bit is called the discriminent.

1) If this is positive we get two answers due to the $\pm$.

2) If this is zero we get one answer because $+0 = -0$.

3) If this is negative we get no answers because we can't square root a negative number.

(i) It intersects once

So $b^2 - 4ac = 0$

$k^2 - 4 \times 3 \times 3 = 0$

$k^2 - 36 = 0$

$k^2 = 36$

$k = \pm 6$

(ii) It has no roots so

$b^2 - 4ac < 0$

$2^2 - 4 \times a \times -4 < 0$

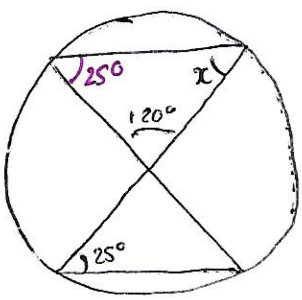
$4 + 16a < 0$

$16a < -4$

$a < -\frac{4}{16}$

$\underline{\underline{a < -\frac{1}{4}}}$

(22) (i) Theorem 3: Angles subtended by the same arc are equal.

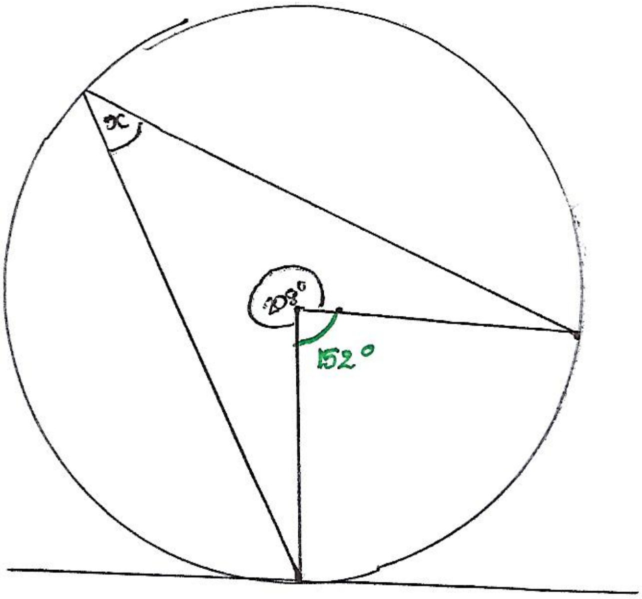


There are 180° in a triangle so

$120 + 25 + x = 180$

$x = 180 - 120 - 25 = \underline{\underline{85°}}$

(ii)



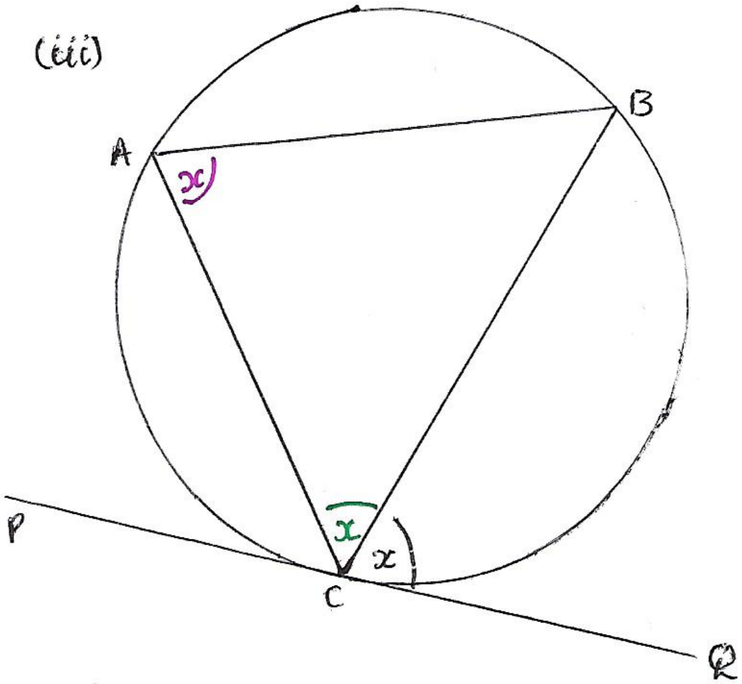
360° around a point.

so $360° - 208° = 152°$

Theorem 1: The angle subtended by an arc at the centre is twice the angle subtended at the circumference.

$x = \frac{152}{2} = 76°$

(iii)



If BC bisects ABQ then

ACB = BCQ

Theorem 8: Alternate segment theorem

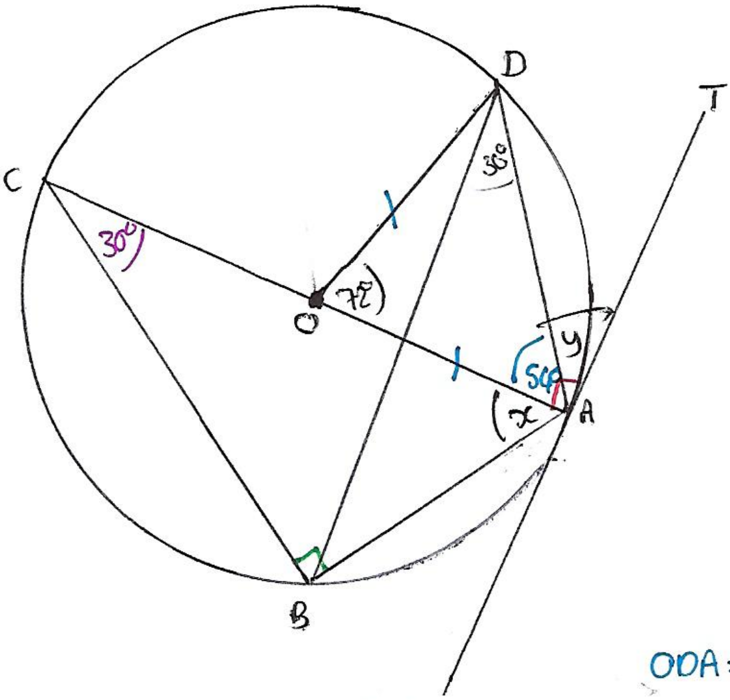
So BAC = BCQ

Since BAC = BCA = x

it's an isosceles triangle

and so AC = BC.

(23)



AC is the diameter. The angle in a semi circle is 90°.

BCA = BDA because angles subtended by the same arc are equal (theorem 3).

Angles in a triangle = 180°

$x = 180° - 90° - 30° = 60°$

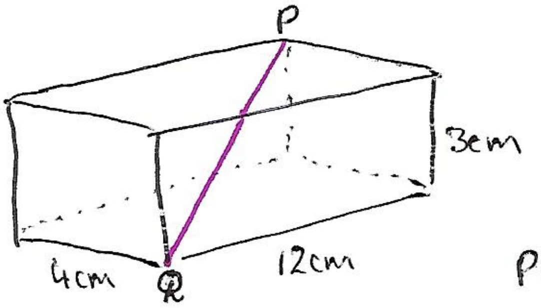
OC = OD = radius so ODA is an isosceles triangle so

$ODA = OAD = \frac{180-72}{2} = 54°$

OAT is a right angle because the angle between a radius and ~~a~~ tangent is 90°.

So $y = 90° - 54° = \underline{\underline{36°}}$

(24)

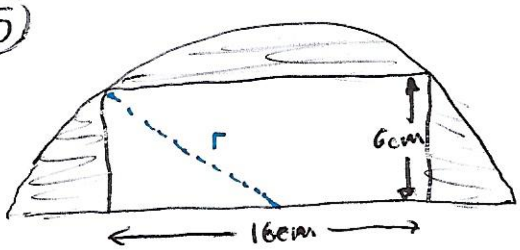


Pythagorus applies in 3D:

$a^2 = b^2 + c^2 + d^2$

$PQ = \sqrt{3^2 + 4^2 + 12^2} = \underline{\underline{13\text{cm}}}$

(25)



Shaded area = Area of semicircle - Area of rectangle

Area of the rectangle $= 6 \times 16 = 96 \text{ cm}^2$

Area of semicircle $= \frac{1}{2}\pi r^2$ ← we don't know r.

We can use Pythagorus to find r:

$r = \sqrt{6^2 + 8^2} = 10 \text{ cm}$

Area of semicircle $= \frac{1}{2}\pi \times 10^2 = 50\pi \text{ cm}^2$

Shaded area $= \underline{\underline{(50\pi - 96) \text{ cm}^2}}$

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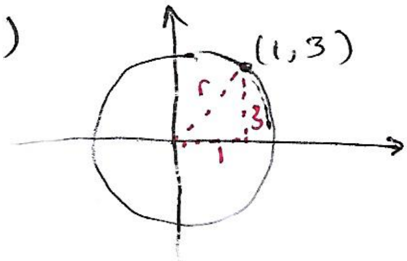
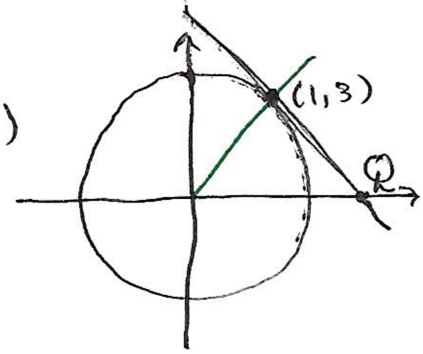
(26) (i)



$r = \sqrt{1^2 + 3^2} = \sqrt{10}$

(ii)



We can work out the gradient of the green line, the tangent's gradient is then the negative reciprocal.

Radius gradient $= \frac{\Delta y}{\Delta x} = \frac{3-0}{1-0} = 3$

Gradient of tangent $= -\frac{1}{3}$

So equation of tangent: $y = -\frac{1}{3}x + c$

Sub in the only point we know $(1, 3)$ to find c:

$3 = -\frac{1}{3} + c \Rightarrow c = \frac{10}{3}$

So equation of tangent: $y = -\frac{1}{3}x + \frac{10}{3}$

Now we know the y co-ordinate of Q is 0.
So sub in and solve for x co-ordinate.

$0 = -\frac{1}{3}x + \frac{10}{3}$

$\hookrightarrow \frac{1}{3}x = \frac{10}{3}$

$\hookrightarrow x = 10$

So $\boxed{Q \text{ is point } (10,0)}$

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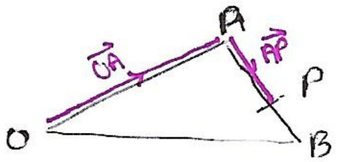
27 We need to work out $\vec{OP}$ ourselves and then compare to what we are given to find k.



To get from O to P ($\vec{OP}$) we need to go up $\vec{OA}$ and then $\vec{AP}$:

$$\vec{OP} = \vec{OA} + \vec{AP}$$

we know $\vec{OA}$ because we are told it's 2a.

↳ $\vec{OP} = 2a + \vec{AP}$

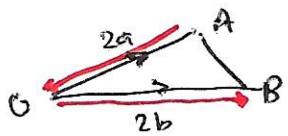
we are told that $AP : PB = 5 : 3$ so

$$\vec{AP} = \frac{5}{8}\vec{AB}$$

because P is $\frac{5}{8}$th of the way along $\vec{AB}$

↳ $\vec{OP} = 2a + \frac{5}{8}\vec{AB}$

so now we need to work out what $\vec{AB}$ is:



The way to get from A to B (using sides we know the length of) is $\vec{AO}$ then $\vec{OB}$.

$\vec{OB} = 2b$ as we are told.

$\vec{AO} = -2a$ because we go in the opposite direction to the arrow so:

$$\vec{AB} = -2a + 2b$$

↳ $\vec{OP} = 2a + \frac{5}{8}(-2a + 2b) = 2a - \frac{5}{4}a + \frac{5}{4}b$

$= \frac{3}{4}a + \frac{5}{4}b = \frac{1}{4}(3a + 5b)$

so $\underline{k = \frac{1}{4}}$

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