Functions

(11) (a) To find the inverse of a function, swaps the x's and y's and then rearrange to make y the subject of the equation.

$f(x)$: $y = 3x + 5$ $\xrightarrow[\text{swap } x \text{ and } y]{}$ $x = 3y + 5$

-5 from both sides $\quad x - 5 = 3y$

divide both sides by 3 $\quad y = \dfrac{x-5}{3}$

$$f(x)^{-1} = \frac{x-5}{3}$$

(b) $f(g(x))$ is what we want to find.

Start with $f(x) = 3x + 5$

notice the difference between what we have, $(f(x))$, and what we want, $(f(g(x)))$. We've got an 'x' where we want a '$g(x)$'.

So replace every x with $g(x)$.

$$f(x) = 3x + 5$$

$$f(g(x)) = 3(g(x)) + 5$$

We know what $g(x)$ is : $g(x) = 2(x-1)$

$$f(g(x)) = 3(2(x-1)) + 5$$

$$= 3 \times 2 \times (x-1) + 5$$

Now just simplify

$$= 6(x-1) + 5$$

$$= 6x - 6 + 5$$

$$fg(x) = 6x - 1$$

(c) As before, $gg(x) = g(g(x))$, so start with $g(x)$.

$$g(x) = 2(x-1)$$

Now we swap x with $g(x)$.

$$g(g(x)) = 2(g(x) - 1)$$

And we know $g(x) = 2(x-1)$ so sub that in.

$$g(g(x)) = 2((2(x-1)) - 1)$$

$$= 2(2x - 2 - 1)$$

$$= 2(2x - 3)$$

$$gg(x) = 4x - 6$$

(12) We need to work out $fg(x)$ from $f(x)$ and $g(x)$ and then compare that to the $fg(x)$ we are given to find c and d.

$$fg(x) = f(g(x))$$

Start from : $f(x) = 2x + c$

Sub $g(x)$ for x: $f(g(x)) = 2g(x) + c$

We know $g(x) = cx + 5$: $f(g(x)) = 2(cx+5) + c$

$$= 2cx + 10 + c$$

Now compare that with $fg(x) = 6x + d$

$$2cx + 10 + c = 6x + d$$

On the right hand side we have $6x$ so we must have $6x$ on the LHS. On the LHS we have $2cx$.

So;

$$2cx = 6x$$

$$\hookrightarrow 2c = 6$$

$$\hookrightarrow c = \frac{6}{2} = 3$$

Similarly on the right we have '$+d$' (our not times x number). On the LHS we have '$+10+c$'. So

$$10 + c = d$$

we already found that $c = 3$.

$$10 + 3 = d$$

$$13 = d$$