Simultaneous Equations

(i) The most consistently successful way to solve simultaneous equations is to make one unknown the subject of the simplest equation and then substitute it into the equation (s).

Our equations:

① $2x - 4y = 19$ and ② $3x + 5y = 1$

Both are equal in simplicity so it doesn't matter which equation we choose to rearrange so we'll use the first one.

$2x - 4y = 19$ → Add $4y$ to each side → $2x = 4y + 19$

→ Divide both sides by 2 → $x = 2y + \frac{19}{2}$

It also doesn't matter which unknown we make the subject, x was chosen here but we could have used

$y = \frac{x}{2} - \frac{19}{4}$ instead.

Now sub our equation for x into equation 2.

$3x + 5y = 1$ subbing in $3(2y + \frac{19}{2}) + 5y = 1$

Now expand and solve for y.

$(3 \times 2y) + (3 \times \frac{19}{2}) + 5y = 1$

$6y + \frac{57}{2} + 5y = 1$

$11y + \frac{57}{2} = 1$

$11y = 1 - \frac{57}{2} = -\frac{55}{2}$

$y = -\frac{55}{22} = -2.5$

Finally by subbing our value of y ($= -2.5$) into our equation for x we can find the value of x.

$$x = 2y + \frac{19}{2} = (2 \times -2.5) + \frac{19}{2} = \underline{4.5}$$

So our answer is $x = \underline{4.5}$ and $y = \underline{2.5}$

It's always best to check your answers using the original equations:

$(2 \times 4.5) - (4 \times -2.5) = 19$ ✓

$(3 \times 4.5) + (5 \times -2.5) = 1$ ✓

So our answer is right.

(iii) our equations are

① $y = x - 3$ and ② $y = 2x^2 + 8x - 7$

Equation 1 is clearly simpler and y is already the subject of the equation so we have the first step done already - so now sub that in to equation 2.

$y = 2x^2 + 8x - 7$ sub in → $x - 3 = 2x^2 + 8x - 7$

Now solve for x.

$x - 3 = 2x^2 + 8x - 7$

↳ $0 = 2x^2 + 8x - 7 - x + 3$

↳ $0 = 2x^2 + 7x - 4$

↳ $0 = (2x - 1)(x + 4)$

$x = \frac{1}{2}$ or -4

I've factorised to find x but if you find that tricky, use the quadratic formula.

Now we have x we can sub in to find y. As we have 2 values of x we will get 2 values for y.

When $x = \frac{1}{2}$

$y = x - 3$

$y = \frac{1}{2} - 3$

$y = \underline{-2.5}$

When $x = -4$

$y = x - 3$

$y = -4 - 3$

$y = \underline{-7}$

So our answer is:

When $x = \frac{1}{2}$, $y = -2.5$ and when $x = -4$, $y = -7$

Check our answers using original eqn $y = 2x^2 + 8x - 7$

Set 1 $x = \frac{1}{2}$, $y = -2.5$

$-2.5 = (2 \times (\frac{1}{2})^2) + (8 \times \frac{1}{2}) - 7 = \frac{1}{2} + 4 - 7 = -2.5$ ✓

so we are right

Set 2 $x = -4$, $y = -7$

$-7 = (2 \times (-4)^2) + (8 \times -4) - 7 = 32 - 32 - 7 = -7$ ✓

(iii) our equations are

① $5x - y = 5$ and ② $2y - x^2 = 11$

Equation 1 is clearly simpler so make x or y the subject of the equation (y is probably easier because it's on it's own where as x is multiplied by 5).

$5x - y = 5$ → add y to each side → $5x = 5 + y$

→ -5 from each side → $5x - 5 = y$

Now sub that in to equation 2.

$2y - x^2 = 11 \rightarrow 2(5x - 5) - x^2 = 11$

$\rightarrow 10x - 10 - x^2 = 11$

$\rightarrow x^2 - 10x + 21 = 0$

$\rightarrow (x - 7)(x - 3) = 0$

So $x = 7$ or 3

Use these values of x to find the values for y.

$y = 5x - 5$

$y = (5 \times 7) - 5$

$y = 35 - 5 = \underline{30}$

$y = 5x - 5$

$y = (5 \times 3) - 5$

$y = 15 - 5 = \underline{10}$

So when $x = 7$, $y = 30$ and when $x = 3$, $y = 10$.

Check our answers using original eqn $2y - x^2 = 11$

Set 1 $x = 7$, $y = 30$

$11 = (2 \times 30) - 7^2 = 60 - 49 = 11$ ✓

Set 2 $x = 3$, $y = 10$

$11 = (2 \times 10) - 3^2 = 20 - 9 = 11$ ✓

so we are right

(16)

A = The price of an adult ticket.

C = The price of a child ticket.

We are told $3A + 4C = 23$ Equation (1)

and $A + 5C = 15$ Equation (2)

These are our simultaneous equations. We get 1 mark setting them up like above from the info we are given.

Both are equally simple equations but equation 2 has 'A' already on it's own (not multiplied by anything).

$A + 5C = 15$ $\xrightarrow{-5C \text{ from both sides}}$ $A = 15 - 5C$

Now sub that in to equation 1.

$3A + 4C = 23 \Rightarrow 3(15 - 5C) + 4C = 23$

Now solve for C.

$(3 \times 15) - 3 \times 5C + 4C = 23 \Rightarrow 45 - 15C + 4C = 23$

$45 - 11C = 23 \Rightarrow 22 = 11C \Rightarrow \underline{C = 2}$

Now used this to find A

$A = 15 - 5C = 15 - (5 \times 2) = 15 - 10 = \underline{5}$

Check our answers,

$(3 \times 5) + (4 \times 2) = 15 + 8 = 23$ ✓

$5 + (5 \times 2) = 5 + 10 = 15$ ✓ so we are right

So adult tickets cost £5 and child tickets cost £2.